Dynamical hierarchies in fundamental physics

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Abstract
A new process oriented physics is being developed at Flinders University. These ideas were initially motivated by deep unsolved problems in fundamental physics, such as the difficulty of quantizing gravity, the missing arrow of time, the question of how to interpret quantum mechanics, and perhaps most importantly, a problem with the very methodology of our fundamental descriptions of the Universe. A proposed solution to these problems, Process Physics, has led to what can be viewed as a hierarchical model of reality featuring a Universe that exhibits behaviour very reminiscent of living systems. A core element of this theory will be discussed.

Modelling Reality
The concepts of emergence, hierarchies and hyperstructures as discussed by Baas (Baas 1992), provide us with a powerful framework for the study of complex systems. However it has been noted (Baas 1992; Mayer & Rasmussen 1998) that the generation of dynamical hierarchies from a formal system is not trivial. In particular the generation of structures with order higher than two is difficult to achieve. It is the aim of the present discussion to present a new dynamical hierarchy that shows strong indications of having at least third-order structure. This hierarchy forms an essential part of a quantum theory of gravity that is currently under development (Cahill 2002) within a new framework called Process Physics. We shall begin with a brief consideration of some key issues faced by theoretical physicists today, in an attempt to justify the radical shift in thinking that is made in this emerging quantum theory of gravity.

Since the time of Newton the aim of physics has been to capture a context free, formal model of reality, that is, describing the Universe in terms that are independent of an observer. This method has been remarkably successful, yielding rich results, many of which have been experimentally verified to a high level of accuracy. However, there are strong arguments suggesting that such a research paradigm may be fundamentally flawed. Perhaps the most famous example is Gödel’s theorem (Nagel & Newman 1995), which suggests that such a programme may lead to an incomplete or inconsistent mathematical model of any system complex enough to be a candidate description of the Universe. While the potential impact of this result in the field of physics is still being debated (Casti & Karlovist 1996), Chaitin has extended this result, showing that Gödel’s unprovable truths are essentially random in character — they are incompressible. (Chaitin 1990; 1999a; 1999b) This result has new ramifications for physics — can these random, incompressible truths be captured by physical models?

Even if these foundational issues are not considered important, a number of apparently unsolvable problems have begun to emerge in the field of physics during the last century. For example, a successful theory of quantum gravity has yet to be discovered. Our model of time as a simple one-dimensional manifold has led to some very well known problems such as the missing arrow of time (Price 1996) and the inadequate description of the present moment effect. Even the interpretation of quantum mechanics remains mysterious, with a number of competing alternative interpretations (Bub 1997; d’Espagnat 1995; Giulini et. al. 1996; Omnes 1994; Percival 1998) each leading to a profoundly different understanding of reality.

The methodology of the modelling used in fundamental physics is also problematic. Our description of reality has been steadily increasing in it’s detail, from atoms to protons, electrons and neutrons to the current family of ‘fundamental particles’; quarks, leptons, mesons etc. However this progression of theories has yet to end. A number of new contenders for the fundamental entities of physics are starting to emerge; strings (Sen 1998) and loops (Rovelli 1998) and preons (Marshak 1998). Is fundamental physics heading towards a regress?4 At some stage an explanation of how a particle can be understood as truly fundamental must be proposed, but in postulating a particle as fundamental we have lost our ability to explain that particle. A new methodology is required.

4This problem has been independently recognised by a number of different researchers, such as Wheeler (Wheeler 1986).
Modelling Life

This problem of methodology could be considered unique to the theoretical physicist. Usually in constructing a hierarchy there is a set of predefined objects that can be legitimately used in that model; DNA and cells in biology; people in sociology; even in mathematics where a set of predefined axioms are constructed by a mathematician, we still have an explanation for the origin of these units namely the mathematician. Are the problems faced by theoretical physicists at all relevant to the rest of science?

The answer to this question must depend upon how reducible we believe the phenomenon of life to be. Is it possible, at least in principle, to describe any naturally occurring system as a set of physical equations, or should we see science as a set of theories, describing successive levels of reality which are nearly decomposable into the ones below? (Simon 1973) As yet, no definitive answer to this question has been provided, but we can surmise that if there were some aspect of reality essential to the description of a living system that was not well described by physical theory then the case of the strong reductionist would be weakened. Can we find such a phenomenon?

The problem of time mentioned in the previous section may provide us with just such an example. There is a very real sense in which the notion of experience is missing from the dynamics of physics, where an external mechanism must be used to model the notion of ‘now’. Consider the following statement by Carnap (Schlipp 1963):

*Once Einstein said that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seems to him a matter of painful but inevitable resignation. I remarked that all that occurs objectively can be described in science: on the one hand the temporal sequence of events is described in physics; and, on the other hand, the peculiarities of man’s experiences with respect to time, including his different attitude toward past, present and future, can be described and (in principle) explained in psychology. But Einstein thought that scientific descriptions cannot possibly satisfy our human needs; that there is something essential about the Now which is just outside of the realm of science.*

This statement describes the way in which our experience of the present moment is missing from physics. Traditionally time is modelled as a one-dimensional manifold, but there is no processing inherent in this model. A ruler provides us with a straight-forward example of a one-dimensional manifold, and we can examine the missing processing of physics through the use of a ruler metaphor of time. The standard way to claim that an event occurred at some time \( t \) is by the addition of a finger on our ruler, pointing to the mark \( t \) on the ruler, but the finger is external to the ruler. The flow of time is captured by running a finger along the ruler, but this flow is external to the ruler. Our experience of this flow is very real, but it is not explained within the field of physics which is concerned solely with the ruler.

While it is possible for the physicist to claim that our undeniable experience of the present moment is a purely psychological phenomenon, this would provide us with the example that we are looking for. However, the fact that physics does not at present describe the present moment does not mean that it cannot in principle. Perhaps it is possible to modify physics so as to model this phenomenon in addition to the currently well understood phenomena of physics.

The standard ‘ruler’ model of time is very much object based. However our experience of time suggests that it is more of a process that we experience. This provides us with a valuable starting point in our attempt to modify physics — can we construct a process-type physics?

Process Modelling

The traditional methodology of the physicist could be described as object based, it centres around some set of fundamental objects and then attempts to describe their dynamics. Perhaps the best known counterexample to this methodology was provided by Leibniz in his dispute with Newton whose description of absolute space he considered inappropriate. Leibniz attempted to construct a relational model of space (Leibniz 1989) but encountered insurmountable problems in this attempt, many of which can be traced to this same object based problem. However, his ideas are very suggestive. Is there a way in which a successful relational model of the Universe can be constructed? In order to solve this problem of object driven methodologies, a subtle bootstrap mechanism has been proposed. We shall first construct a relational
model, and then discuss the bootstrap mechanism in the next section.

Considering a set of $M$ nodes, we assume that they are connected in some way, with a connection strength between node $i$ and node $j$ given by the value $B_{ij}$ (see figure 1). We shall represent this set of relational values as a square antisymmetric matrix $B$. Notice that while the pitfalls of object driven methodologies have been explicitly recognised above, we are still driven to talk in terms of nodes. It is very difficult to leave an object methodology behind. As discussed above however, any system positing such a priori objects cannot be considered fundamental when modelling the Universe, as the explanation of these objects must lie outside the system being modelled. A solution to this dilemma arises if we recognise that the nodes can in turn be modelled as a system of nodes, connected with weaker $B_{ij}$ values. It becomes apparent that all nodes can be thought of as composed of collections of nodes in turn, and in particular that the start up nodes can be viewed as names for subnetworks of relations. This result is ensured if the constructed system exhibits self-organised criticality (SOC), which enforces a fractal structure on the system (Bak, Tang & Wiesenfeld 1998).

An initial connection with dynamical hierarchies can be made if we consider the nodes at one arbitrary level as a family of structures of first order (Baas 1992). The next section will be devoted to discussing the emergence of higher order structures from this initial family within the Process Physics paradigm. We will return to dynamical hierarchies at a later point where Process Physics will be discussed explicitly within the context of dynamical hierarchies.

![Figure 1](image-url)

Figure 1: (a) Nodes $i$ and $j$ are considered connected if they have a non-zero $B$ value. Arrows indicate the sign of the $B_{ij}$ value. (b) Self-links are considered internal to a node so $B_{ii} = 0$.

The SOC requirement is intrinsically linked with a new notion of time in this system. This is because the relational fractal structure is generated by a noisy non-linear iterative map displaying SOC behaviour. Thus in attempting to construct a model that does not postulate a priori fundamental objects, we find the need to introduce a time-like process. In contrast, standard physics with its use of a priori objects is linked with the standard geometrical model of time and the associated problems mentioned above.

The particular map used was suggested by the Global Colour Model of quark physics (Cahill & Gunner 1998). Stripping away the space time and quantum number indices (Cahill & Klinger 1998), and adding a noise term suggested by the stochastic quantisation procedure of Parisi and Wu (Parisi & Wu 1981) leads to the following iterative process (Cahill & Klinger 1999)

$$B_{ij} \rightarrow B_{ij} - a(B + B^{-1})_{ij} + \omega_{ij},$$

where

$$i, j = 1, 2, \ldots, 2M \quad \text{and} \quad M \rightarrow \infty.$$  

The term $\omega_{ij}$ represents an additive noise term. The system can be understood as open because of this term. At each iteration, the noise term creates new $B_{ij}$ links which provides the system with a sense of innovation and contingency. The noise term, when used iteratively in equation (1) is responsible for the notion of time that arises in the model. The dynamics are irreversible, with one particular past, which can be recorded as a history, but not relived. Future states of the system cannot be known, however certain sets of ensemble predictions can be made. In this sense a processing notion of time is captured by the system, with a markedly different ontology from the time of standard physics. Note also that this modelling of time is far more appropriate in the context of living systems, providing a sense of contingency and dynamism. We shall discuss this point more in the final section of this paper, but it is clear that this model provides us with the more satisfactory description of time as internal to physics that we went in search of.

The nonlinear matrix inversion term also performs a critical role in the system. It causes separate structures brought into existence by the noise term to link up, modelling a process of self-assembly. It is interesting to examine the dynamics of this process in detail.

The system can be started with $B \approx 0$ which represents the absence of any significant relational information. Sparse matrices can be decomposed into sets of independent matrices, and it is possible to use this fact to extract more information about the system that emerges under successive iterations of equation (1). Assuming that the large $\omega_{ij}$ arise with fixed but very small probability $p$, the geometry of the structures formed can

\[^4\text{Antisymmetry ensures that }B_{ij} = 0\text{ thus avoiding explicit node self connection. The internal structure of nodes will be incorporated shortly.}\]

\[^5\text{$B$ is assumed to be a very large } (\to \infty) \text{ matrix.}\]
be revealed by studying the probability distribution of minimal spanning graphs with $D_k$ nodes and $k$ links from an arbitrary node $i$ where $D_0 = 1$ (see figure 2). This probability distribution is given by (Nagels 1985)

$$P[D, L, N] \propto \frac{p^{D_1}}{D_1!} \ldots \frac{p^{D_L}}{D_L!} \prod_{i=1}^{L-1} (q \sum_{j=1}^{D_i} x_j^{D_{i+1}} (1-q^{D_i})^{D_{i+1}})^{L-1}$$

where $q = 1 - p$, $N$ is the number of nodes and $L$ is the maximum depth from node $i$. The most likely pattern can be found by numerically maximising $P[D, L, N]$ for fixed $N$ with respect to $L$ and $D_k$. This procedure has been performed (Cahill & Klinger 1999), and the following results are from that analysis.

Figure 3 shows the set of $D_k$ (distance distribution) values obtained from one of these numerical experiments, where the log of the probability of a large noise value is set at $\log_{10} p = -6$, and the number of nodes is fixed at $N = 5000$. Also shown in the figure is a curve

$$D_k \propto \sin^{d-1} (\pi k/L)$$

with best fit to the data when $L = 40$ and when the dimensionality of the fit, $d = 3.16$. This curve is obtained from the surface area of a $n$-dimensional sphere. Figure 4 shows the range in $d$ for fixed $N = 5000$ and varying $p$ values. We see that for $p$ below some critical value $\log_{10} p < -5$, $d \approx 3$.

This indicates that the connected nodes have a natural embedding in a $S^3$ hypersphere, which is very suggestive of the 3-dimensionality of space. Thus this model goes some way towards predicting the observed three-dimensional structure of space, a feature that is usually assumed in physics. Notice that the nodes are not exactly embeddable (which would require $d = 3$), there is a proportion of extra links. This is a key observation which forms the basis of a theory of matter as a quantum

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6As represented by the $B$ matrix.
foam that is currently under development (Cahill 2001; 2002), which will not due to space considerations be discussed here.

While they are not the only structures generated by the system, their maximum likelihood makes these structures the most common. This discussion will centre on the trees for the sake of analysis but a more comprehensive theory is currently being developed. Splitting the large $B$ matrix into its constituent independent sub-matrices $B_{sub}$ we realise that each is almost singular (det$(B_{tree}) \approx 0$) but that the noise term ensures extra $B_{ij}$ terms which lead to a small valued determinant. Upon inversion of the $B$ matrix, this small valued determinant ensures that the next iteration will lead to new large valued $B_{ij}$ entries (depending upon the specific $\omega_{ij}$ noise terms). Hence tree structures are sticky, at each iteration cross-links form between the structures which act to join them, and to produce larger structures.

The nonlinear term is a self referencing term, all elements of $B$ are required in order to compute the next value of each $B_{ij}$ element from the previous iteration. This term can be seen to incorporate a weak notion of internal self-observation into the system. The noise term $\omega_{ij}$ limits this self-referencing of the system, providing the system with a sense of innovation.

Thus, under the influence of the iterator (1) the system can be seen to ‘grow’ with an steady increase in relational structure. The openness of the system provided by the noise term is essential here, as otherwise the system could be seen as violating the 2nd law of thermodynamics.

Hyperstructure

There is a sense of emergence in this system. Structures appear to be forming which are not explicitly present in either the update equation (1) or the original very simple relational structure of nodes and connections. An examination of this claim within the framework of dynamical hierarchies helps to clarify, in less technical terms than were used in the previous sections, this aspect of the behaviour of the system.

An essential aspect of the dynamical hierarchies framework is provided by the notion of observation (Baas 1992). In order to register the emergence of new structures or entities within a system it is necessary to identify a mechanism capable of observing those entities. This notion is not present in Process Physics, as we have been interested in generating a system from very simple initial conditions, however it is possible to identify observational mechanisms (represented by $O$ in what follows) through the adoption of what might be regarded as an external perspective.\footnote{Most of the analysis applied in this discussion falls under this external status. The majority of analytical tools appear to be of an external nature, but this does not imply that}

A dynamical hierarchy can now be generated if we consider the original nodes used in the construction of the relational structure to be a very simple family of $M$ first-order structures $\Xi^1 = \{ \Xi^1_r : r = 1, 2, \ldots, M \}$, where the first-order observational mechanism $O^1$ is defined as membership in the set $\Xi^1$. Now, under the influence of equation (1) which we consider to be an update functional $R$, we find that these nodes join up, forming a second order structure:

$$\Xi^2 = R(\Xi^1_r, O^1, B_{ij}) \quad i, j = 1, 2, \ldots, M$$

where $B_{ij}$ is the connection strength between node $\Xi^1_r$ and node $\Xi^1_i$ that appears in the $B$ matrix in the previous sections. The family of second order structures consists of the maximum likelihood structures discussed above that are embeddable in the three-dimensional hypersphere ($E^3$), along with a variety of other structures that have not as yet been classified. However, considering the space-type structures alone, we can define the property of being embeddable in $S^3$, ($E^3$) as a second order computationally emergent property,

$$P_{E^3} \in O^2(\Xi^2).$$

That is, this property is not present in the simple set of nodes that we started with,

$$P_{E^3} \notin O^2(\Xi^1).$$

Only under the influence of the update functional (1) did this extra aspect of the behaviour of the nodes become apparent.

The analysis of the remaining structure of this system is very complex and only in the preliminary stages, but it is expected that stable patterns identifiable as third order structures will emerge in the system. The full theory of this process is currently under development, but the general argument can be sketched out. We can somewhat artificially classify the structures in the system as being either exactly embeddable in $S^3$ (the $E^3$ structures), or non-embeddable structures which we will term defects ($D$). It is expected that the system contains stable defects, which we shall term Topological Defects ($TD$). The stability of these structures will be structural, and constitutes a new third order observation mechanism within the system $O^3$. Thus it is expected that the stable $TD$’s will form a set of $K$ new emergent structures $\Xi^3 = \{ \Xi^3_v : v = 1, 2, \ldots, K \}$ within the system. Equation (1) ensures a constant updating of their components — rather like the units of an organism which are being constantly regenerated. These $TD$’s (if rigorously identified) would be third order structures in the hierarchy. That is

$$P_{TD} \in O^3(\Xi^3_v), \quad P_{TD} \notin O^3(\Xi^2).$$

an entity exists external to the Universe, merely that these external modes of analysis are simpler to implement.
because their emergent stability within the system is defined by their structural stability, i.e. it constitutes a new observation mechanism. It is expected that $TD'$s made up of aggregations of $E_{33}$ structures will occur in this system, not themselves having the $E_{33}$ property.

So far a third order hierarchy has been identified (but not rigorously at the third order).

$$\text{nodes} \quad > \quad \frac{S^3 + D}{O^3(\mathbb{Z}^3)} \quad > \quad \frac{\text{stable } TD}{O^3(\mathbb{Z}^3)} \quad > \ldots$$ (9)

It is expected that more structure will emerge from the system. This is because the identified stable structures of this model have a deep connection with the more standard object driven methodologies of particle physics which will be mentioned in the next section. Due to space constraints this connection will not be examined in any detail, but a link to the standard model of particle physics means that the standard structures of particle physics can be expected to emerge from this theory.

The system presented is rather minimal when compared to the complex task of modelling living systems. This is to be expected. While a surprising amount of structure appears to emerge from this system, the lack of objects in the model plays a key role in limiting the storability of the information of the system. For example, consider biological modelling where complex information can be stored in DNA molecules and their enveloping cells. This provides an a priori fundamental set of information the interactions of which must be incorporated into any model. It also allows for the creation of extremely complex structures, as the system can use this prestored information to generate new structures. We expect that with a strong argument for the stability of the $TD$’s the Process Physics system may become capable of storing such complex information, and hence generating a very rich set of structures undergoing complex interactions. This is clearly a priority for future research.

It is interesting to ask how much higher order structure this system is capable of generating. The ansatz for dynamical hierarchies, (Rasmussen 2001) claims that:

*Given an appropriate simulation framework, an appropriate increase of the object complexity of the primitives is necessary and sufficient for the generation of successively higher-order emergent properties through aggregation.*

This claim has quite profound implications for our argument that physics may in principle be capable of explaining at least the general behaviour of higher level sciences. Is the ansatz a reasonable one? While the system presented here appears to be very minimal, with very simple first order structures, the update equation (1), being derived from Quantum Chromodynamics presumably has an large amount of intrinsic information. In addition to this, it appears that the information originally stripped away from equation (1),

The Minkowski measure of space-time, and the quantum number indices.
theory. New results suggest that this PRF can be probed and is in agreement with the speed given by the COBE CMBR dipole moment (Cahill & Kitto 2002; Cahill 2002). Experiments are currently underway to detect both this frame and a predicted gravitational inflow of space. A PRF allows for the possibility of a realistic interpretation of quantum phenomena, which is currently under construction within the process theory.9

The nature of fundamental models
Finally, we should note that this concrete model suggests a possible list of properties that must be possessed by any physics model if it is to be considered fundamental:

- it must exhibit SOC as this effect is critical to the bootstrap mechanism. It is only with the use of the bootstrap that we were able to avoid the problem of identifying some ‘fundamental’ physical object.
- This implies that fundamental models must have a dissipative nature (Nichols & Prigogine 1997).
- They should also be noisy and nonlinear as these characteristics serve to give a structure that grows in complexity and size.

The conjecture can be made that there will be a class of such models, all able to consistently model the Universe in the way that the model presented here does. In fact the lack of such a class of models would be a fatal flaw in the theory as it would mean that the organisation exhibited by the model is in some sense a relic of the model, providing equation (1) with too much ontological significance. Contrary to some at present popular ideas (Wolfram 2002) it is not expected by us that the Universe really is some sort of computer, rather that the results of Goedel and Chaitin mentioned in the introduction are significant to the programme of physics and that at a fundamental level there are very deep reasons why the Universe cannot be known in the traditional sense expected by physicists (Cahill 2002).

Living Systems
The observation that living systems exhibit many of the features present in this system can be made. For example, both living systems and the Process Physics model are open,10 both systems have a dissipative nature, and are dependent upon a constant processing rather than

9The theory under construction utilises the Quantum State Diffusion model of Gisin and Percival (Percival 1998).
10Although the openness of living systems is simpler to understand — living systems have an obvious environment, while the openness of Process Physics arises as a result of the noise term alone.

References
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